The Tangent Circle

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Abstract

The purpose of this project was to construct a formula to find the tangent circle between any given three points that lie on a curve. We did this by using basic geometric and algebraic principles, as well as some reformed calculus methods. Using said principles, we were able to come up and experiment with a functioning formula. Through extensive experimentation of decreasing and increasing the distances between points, we discovered numerous different properties of tangent circles.

What is a Tangent Circle?

A tangent circle is a much like a tangent line, with some slight differences. A tangent circle is different in that it passes through any three given points on a graph, as opposed to two. As the distance between these three points decreases to zero, a circle tangent one of these points in formed on the graph, with a relative center point.

The Development Process

In our attempt to construct a formula for a tangent circle using only algebra, geometry, and calculus, we started with what we knew and found relevant relations between the concepts we understood that can be applied uniformly to any set of points. For example: for any three points we selected, (labeled with variables), we found the midpoints between two sets of adjacent points. After taking the perpendicular bisector of each midpoint, we found that where the perpendicular bisectors intercepted was our center value for the circle. Then, we found a formula for the (x,y) coordinates of a standard circle center point, and plugged in midpoint and perpendicular bisector formulas; substituting specific values for out chosen variables.

The Formula

2 $f(x) = \sin(x)$

3 $$x_1 = a - \frac{f'(a) + f'(a)^3}{f''(a)}$$

$x_1 = 4.42918532476$

4 $$y_1 = f(a) + \frac{1 + f'(a)^2}{f''(a)}$$

$y_1 = 0.73853881342$

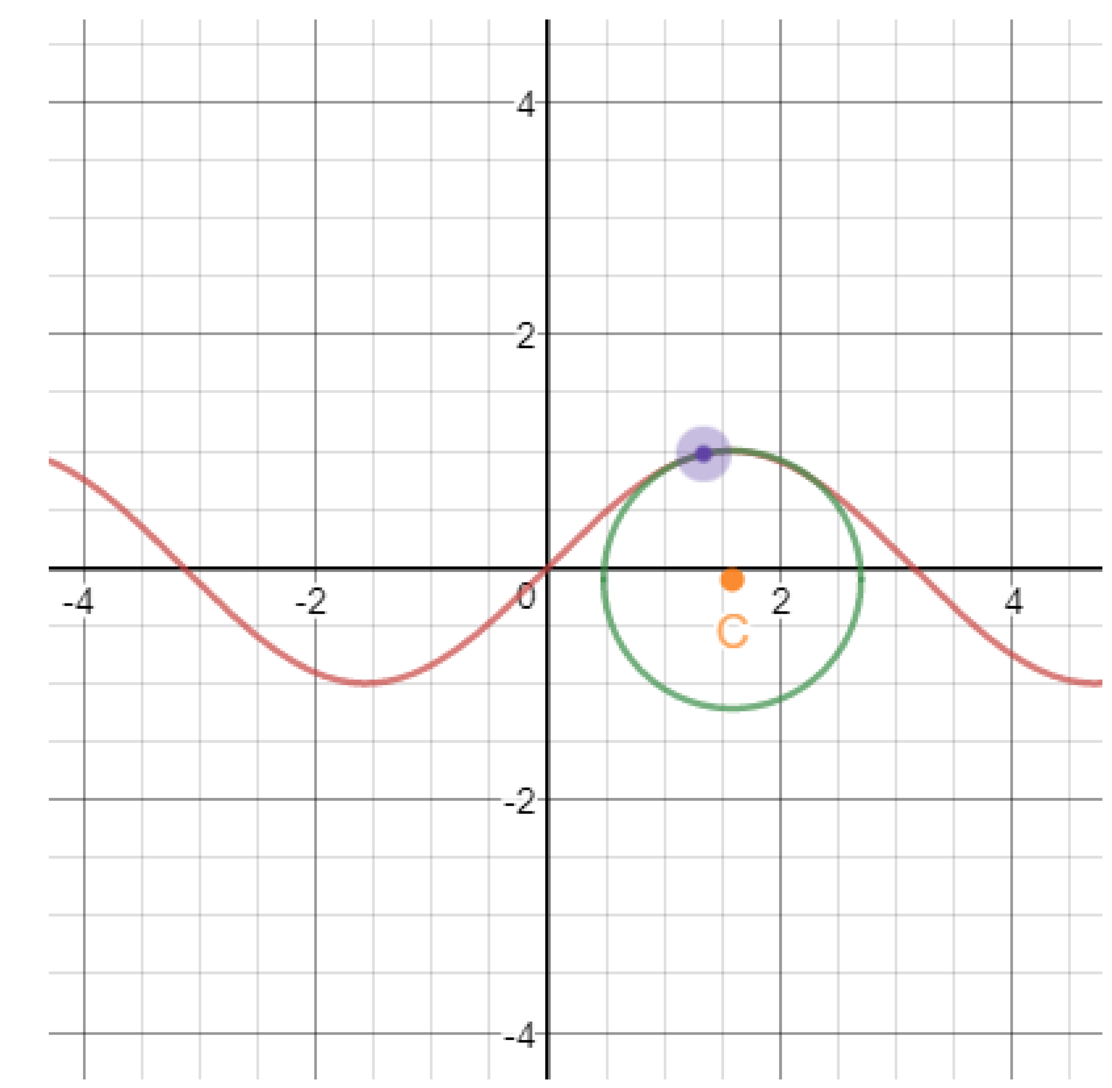
5 $$(x - x_1)^2 + (y - y_1)^2 = (a - x_1)^2 + (f(a) - y_1)^2$$

Data / Observations

After deriving a workable formula and testing it in the online *Desmos* calculator, we made several observations that stood out; here are a few of them:

- When the slope of a graph starts to form a straight line, the tangent circle to that point becomes infinitely large, eventually appearing to be a line
- For graphs with solid curves such as sin(x) and x^2, there are certain values of the three points where the circle perfectly fits the shape of the curve

Results



Conclusion

With the help of Dr. Gubkin, we were able to successfully construct a working formula to find the tangent circle between any given three points of a graph using only algebra, geometry, and calculus. In addition, through experimentation with our final product, we found out several unique aspects of tangent circle as well.

Acknowledgments

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