Mathematical Model of Wind Turbine Power Output

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INTRODUCTION

Day to day, wind speeds vary greatly, but the need for power output does not. Without some way to control and manage the power output driven by the turbines, the energy will waver, which is highly unfavorable. With a mathematical model of control laws however, the power factor can provide the most efficient energy output.

Abstract

This research done by Arantxa Tapia, Gerardo Tapia, J. Xabier Ostolaza, and José Ramón Sáenz, aims to represent a mathematical model of a wind turbine driven by a doubly fed induction machine and models a few examples of real-life performance of these machines. The model demonstrates the operating conditions of the machine both above and below the synchronous speed which provides the ideal power output. This modeling is possible via the use of a double-sided PMV converter, which connects the rotor to the power grid. This researcher utilized MATLAB, SIMULINK, and other machines to arrive at a model. The model of the wind generator is designed to show how a control strategy can be used to manipulate the power factor of the energy that the wind generator supplies. [1]

OBJECTIVES

To develop a mathematical model to analyze the power output of a wind turbine driven doubly fed induction generator. [1]

METHODS

$$\begin{bmatrix} x_a \\ x_b \\ x_c \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & 0 & 1 \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} & 1 \\ -\frac{1}{2} & -\frac{\sqrt{3}}{2} & 1 \end{bmatrix}}_{C^{-1}} \begin{bmatrix} x_d \\ x_q \\ x_0 \end{bmatrix}. \qquad (21)$$

$$\begin{bmatrix} x_d \\ x_q \\ x_0 \end{bmatrix} = \frac{2}{3} \underbrace{\begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix}}_{C} \begin{bmatrix} x_a \\ x_b \\ x_c \end{bmatrix} \qquad (20)$$

$$\begin{bmatrix} v_{sD} \\ v_{sQ} \\ v_{r\alpha} \\ v_{r\beta} \end{bmatrix} = \begin{bmatrix} R_s + sL_s & 0 & sL_m \cos\theta_r & -sL_m \sin\theta_r \\ 0 & R_s + sL_s & sL_m \sin\theta_r & sL_m \cos\theta_r \\ sL_m \cos\theta_r & sL_m \sin\theta_r & R_r + sL_r & 0 \\ -sL_m \sin\theta_r & sL_m \cos\theta_r & 0 & R_r + sL_r \end{bmatrix} \begin{bmatrix} i_{sD} \\ i_{sQ} \\ i_{r\alpha} \\ i_{r\beta} \end{bmatrix}$$

PARAMETER	VALUE
R_s, stator resistance per phase	0.0067 Ω
X_{ls}, stator leakage reactance per phase	0.0300 Ω
n, general turns ratio	0.3806
X_m, mutual reactance	2.3161 Ω
R_r, rotor resistance per phase	0.0399 Ω
X_{lr}, rotor leakage reactance per phase	0.3492 Ω

(Equations[1])

Fig.1: Wind turbine field

RESULTS

- The results of the simulation, obtained by running the wind generator and its control model, correspond with the real life doubly fed induction generators used on a wind farm.
- Although wind speeds vary greatly, the designed control laws can find an ideal power factor for the wind generator.
- This means that the model developed in this paper can provide the most efficient active power to be generated for each wind speed. [1]

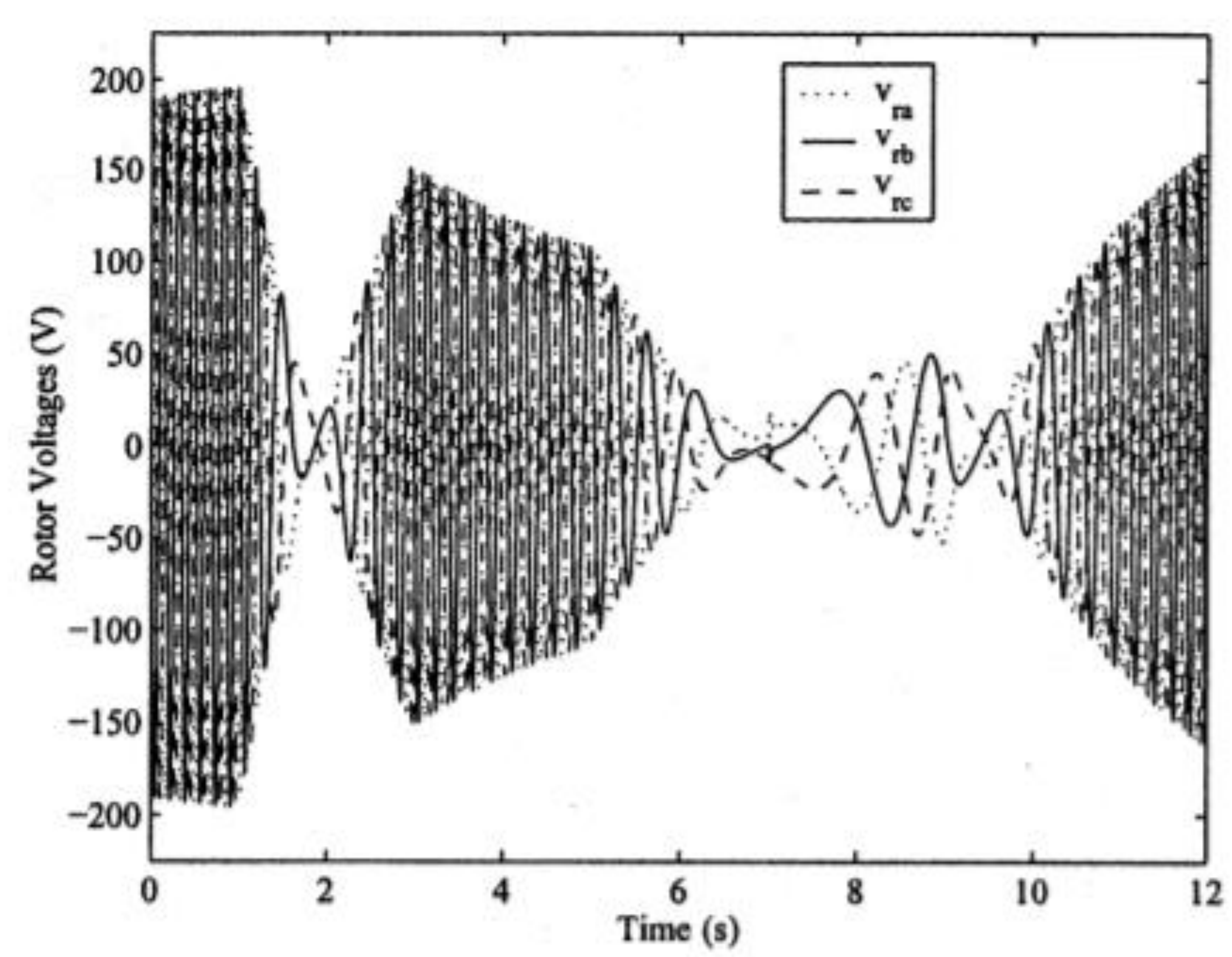

Fig.2: Rotor Voltage v. Time [1]

CONCLUSIONS

The developed model discussed in this paper is proven to be reliable by comparing the simulation to real life data. This model would be an ideal application to modern wind farm generators and would increase the efficiency of the overall energy output. [1]

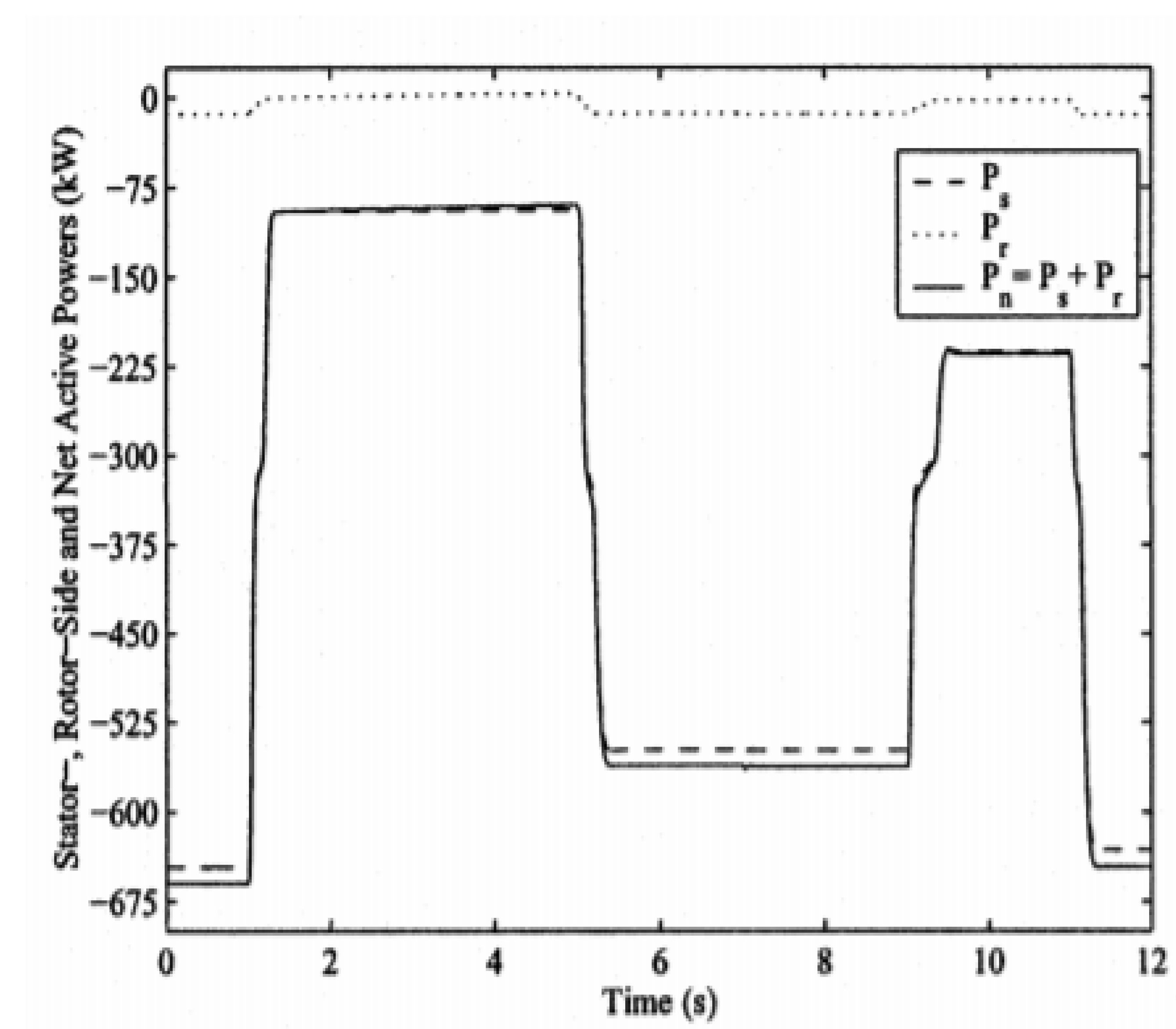

Fig.3: Power Output [1]

FUTURE WORK

While doing this expository research, in the future with more time we would like to use MatLab to input the matrix given as a mathematical model, and further evaluate the power output with various differing graphs.

References

[1] Arantxa Tapia, Gerardo Tapia, J. Xabier Ostolaza, and Jose Ramon Saenz. "Modeling and Control of a Wind Turbine Driven Doubly Fed Induction Generator." *IEEE TRANSACTIONS ON ENERGY CONVERSION*, VOL. 18, 2003, pages 194-204.

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